PhD Research Proposal Form
China Scholarship Council (CSC) - ENS Group

FIELD: Mathematics

Thesis subject title: **Mathematical study of Prandtl's boundary layers**

Name of the French doctoral school : InfoMaths (ED 512)

Name of the Research team : Unité de Mathématiques Pures et Appliquées (ENSL)

Website : http://www.umpa.ens-lyon.fr

Name of the Supervisor : Emmanuel Grenier

Email : Emmanuel.Grenier@ens-lyon.fr

Lab Language : French and English

Research Proposal Abstract :

The question of the behavior of solutions of the Navier Stokes equations as the viscosity goes to zero is one of the most classical and open problems in mathematical fluid dynamics. Its history began in the 19th century with works of Lord Rayleigh and continued in the 20th century with physical studies by Prandtl, Orr, Sommerfeld, Schlichting and Lin amongst many others.

The mathematical study started with T. Kato in the 80's, followed by Caflish and Sammartino, before a series of recent progresses by the supervisers of the current proposal, in collaboration with Y. Guo (Brown University) and T. Nguyen (Penn State University). Part of the corresponding linear problem is now well understood, in particular the lower branch of instability. The nonlinear problem is also completely understood for a particular set of initial data.

This phD would focus on the description of the linearized problem near the upper branch of instability (description of the Green function of the resolvent of the linearized problem, description of the dispersion relation, precise description of the upper branch for various boundary layer profiles). These linear results would then be used to try to understand the non linear instability of generic initial data.

This phD would be a joint phD with D. Bian (Associate Professor, Beijing Institute of Technology, biandongfen@bit.edu.cn).

References :

D. Bian, E. Grenier: Onset of nonlinear instabilities in monotonic viscous boundary layers, *Science China Mathematics*, 2023.
E. Grenier: On the nonlinear instability of Euler and Prandtl equations, *Comm. Pure and Appl. Math.*, 53 (2000), 1067-1091
E. Grenier, Y. Guo, T. Nguyen: Spectral instability of characteristic boundary layer flows, *Duke Math J,* 165(16): 3085 – 3146, 2016
E. Grenier, T. Nguyen: L^infinity instability of Prandtl layers, *Annals of PDE,* 2019
M. Sammartino, R.E. Caflish: Zero viscosity limit for analytic solutions of the Navier Stokes equations in the half plane, *Comm. Math. Phys.*, 192(2), 1998

Type of PhD :

1.Full PhD

- Joint PhD/cotutelle (leading to a double diploma) : YES
- Regular PhD (leading to a single French diploma) : NO

2. Visiting PhD (for students enrolled at a Chinese institution who will be invited to a French institution to carry out a mobility period) : NO